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LINEAR TIME-HISTORY ANALYSIS AS EC8-COMPLIANT DESIGN METHOD: WHAT SEISMIC INPUT SELECTION?

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ABSTRACT

Linear Time-History Analysis (LTHA) is employed as EC8-compliant design method for a regular 12-storey archetype Reinforced Concrete (RC) regular Moment Resisting Frame (MRF) including structural features typically not included such as stairs and considering different ground motions selection options. Firstly, a suite of seven pairs of spectrum-compatible ground motions from the European Strong Motion Database is used; secondly, the spectral-matching procedure for three pairs of ground motions suggested by the recent FEMA P-1050/2015 is adopted. The critical aspect is to find a balanced compromise between control of variability in the suite of ground motions, better suiting design purposes, without losing the opportunity to capture, at the design stage, part of the record-to-record variability in far-field and near-field conditions.
Eurocodes are currently in the phase of review for a second-generation release; this study aims to deal with the gap in the code related to the possibility of designing structures through LTHA.
The study employs a new definition of the Eurocode 8 behavior factor (i.e., strength reduction factor) for force-based approaches. Results of this study point to a paradigm shift in design towards a direct performance-based approach at design-stage. Preliminary comparisons suggest that a balance between the close spectral matching of FEMA P-1050/2015 and the current selection procedure of EC8 should be found for LTHA.

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Linear Time-History Analysis (LTHA) is employed as EC8-compliant design method for a regular 12-storey archetype Reinforced Concrete (RC) regular Moment Resisting Frame (MRF) including structural features typically not included such as stairs and considering different ground motions selection options. Firstly, a suite of seven pairs of spectrum-compatible ground motions from the European Strong Motion Database is used; secondly, the spectral-matching procedure for three pairs of ground motions suggested by the recent FEMA P-1050/2015 is adopted. The critical aspect is to find a balanced compromise between control of variability in the suite of ground motions, better suiting design purposes, without losing the opportunity to capture, at the design stage, part of the record-to-record variability in far-field and near-field conditions.

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Introduction

Linear Time-History Analysis (LTHA) can represent a simple tool for practitioners within the Performance-Based Earthquake Engineering (PBEE) framework, overcoming typical obstacles due to the complexity of nonlinear models and approximate assumptions typical of linear analyses (i.e., modal combination rule, one degree-of-freedom analogy, etc.) [1].

The input is characterized by accelerometric waveforms resulting by seismic input selection. So, it is possible to account for the interaction between the vibration modes of the structure with the typical frequency of earthquakes [2]. The nonlinear version of this analysis is the so-called Nonlinear Time-History Analysis (NTHA) in which the nonlinearity is explicitly accounted for in the model. NTHA is the most rigorous analysis method. Notwithstanding the growth in computer processing power, NTHA is still time-consuming and it needs an appropriate choice of the ground motions, analytical model and acceptance criteria [3;4]. Contrarily, Nonlinear Static Analysis (NSA), also known as Static Pushover Analysis, is less demanding than NTHA and it is nowadays implemented using various approaches in the majority of commercial software packages. However, NSA is not consistently reliable in the case of multi-storey buildings [5], therefore the current professional practice can be improved for both new and existing structures. Also, nonlinear analyses are not generally conceived to be used at design stages so a balanced compromise between accuracy of structural response evaluation and simplicity of design procedure for all the possible practical cases (i.e., high-rise, low-rise, regular and irregular structures) is needed.

Some current design codes, excluding Eurocode 8 (EC8), consider LTHA as an option amongst the seismic methods of analysis and some indications are now provided for LTHA design. Recently, a new LTHA procedure for design [6] has been approved for inclusion in the FEMA P-1050/2015 [7] and ASCE/SEI 7-16 [8]. In the last two codes, the *response spectrum matching* method, based on the non-uniform wavelet-based spectral matching of ground motions with respect to the target spectrum (i.e., code-base spectrum), is explicitly suggested for the input selection, so that LTHA can be used as an alternative to the Response Spectrum Analysis (RSA), with the principal benefit that signs of response effects are preserved and without asking for extensive and time-consuming record selections.

In this preliminary study, a new procedure for design according to EC8 is proposed and differences with respect to the FEMA P-1050/2015 are analyzed for the case of a 3D 12-storey regular Reinforced Concrete (RC) Moment Resisting Frame (MRF) building modelled in OpenSees [9]. Such procedure is proposed as an improvement of the current design practice which is based on RSA as reference method (with particular reference to the case of high-rise structures where higher-modes effect can lead to significant differences with respect to the RSA [10]). In the proposed procedure, the input is selected considering unscaled real ground motions which are well-known to lead to a real seismic response even if very dispersed. This would allow to consider specific situations such as Near-Fault ground motions and fault-rupture directivity. Finally, some relevant aspects such as the application of the behavior factor, P-Delta effects and time-history components combination are discussed.

EC8-Design of the Benchmark Structure

The benchmark structure considered is a 3D 12-storey regular RC-MRF building, located in Pettino, L'Aquila (Italy), and designed through RSA with respect to the Ductility Class High (DCH) prescriptions according to EC8 [11] and EC2 [12], including the specifications of the Italian National Annexes. More details about geometry and materials can be found in [1]. For Life Safety Limit State (LS-LS), the Italian National Annexes suggest that the elastic UHS with 10% probability of exceedance of 50 years (Return Period, T_R = 475 years) must be converted into a design spectrum through the employment of a behavior factor, in this case q = 5.85 for dissipative behavior and regular structural configuration. The benchmark building accounts for some aspects like the presence of staircase, maximum number of storeys for ordinary concrete classes, relevant influence of higher-modes of vibration and significant P-Delta effects.

Ground motions selection for time-history analyses

In RSA, the peak value of the generic mode contribution to the response can be obtained from the Uniform Hazard Spectrum (UHS); subsequently, the peak value of the total response is obtained by combination of the peak modal responses and assuming that peaks are all attained at the same time instants. There are different modal combination rules, but the wider applicable rule is the Complete Quadratic Combination (CQC). Contrarily, LTHA needs a proper selection of ground motions that has to be spectrum-compatible with the UHS. Basically, the record-to-record variability needs to be accounted for in different way according to the final goal of the analysis

[13-15]; being fundamental for the estimation of the probability of failure but allowed to be reduced for mean response estimation.

It is recognized that LTHA cannot be expected to predict the behavior of systems meant to show a nonlinear behavior, and as such it is merely a tool to be used for design. As it is shown later, FEMA P-1050/2015 just requires that a suite of a minimum of three ground motions have to be considered and non-uniformly scaled to get the matching with the UHS. This is aimed at developing a response history procedure that uses a very close input with respect to the conventional RSA.

More in general, there are two main approaches for record scaling: (i) amplitude scaling and (ii) spectral matching. In (i), the ground motion is multiplied by a constant scale factor so that the respective pseudo-acceleration response spectrum and the UHS coincide at a specific period of vibration (generally the fundamental period of the structure), or such that the average of the scaled components from a suite of earthquakes closely matches (within some tolerance) the UHS in a specific range of periods of interest. If the same scale factor is applied to the components of the same earthquake belonging to the suite of earthquakes, the frequency characteristics of the original earthquake are preserved. Experience has shown that not many ground motions are necessary to get the matching with the UHS but an elevated number of them should be used if the record-to-record variability needs to be controlled. The mismatch between the average of the suite and the target spectrum can be quantified by the Root-Mean-Square-Error (RMSE) as shown in Eq.(1), where $Sa_{avg,i}$ and $Sa_{UHS,i}$ are the pseudo-accelerations of the average response spectrum and the UHS at the i^{th} period value and n_p is the number of periods in the specific range of periods of interest.

$$RMSE = \sqrt{\frac{1}{n_p}\sum_{i=1}^{n_p}\left(Sa_{\text{avg,i}} - Sa_{\text{UHS,i}}\right)^2} \tag{1}$$

In (ii), the original ground motion is non-uniformly scaled, by using Fourier transforms or wavelets for example, such that the pseudo-acceleration response spectrum of the matched record closely matches the shape of the target spectrum. It results in a reasonable variation in response among earthquakes of a suite of ground motions. Experience has shown that three pairs of ground motions can provide satisfactory results if the envelope response is considered.

Another important aspect is related to the fact that many codes do not explicitly account for shapes of UHS suitable for pulse-like motions as well. It is well-know that most of the energy in a pulse motion is released in one or two cycles of the velocity-time series and often it produces a high spectral demand at longer periods than ordinary motions [16]. In these cases, it is important to simulate a structural response which implicitly accounts for pulse-like motion effects.

Table 1 summarizes the input selection approaches for EC8 and FEMA P-1050/2015. The current version of EC8, issued in 2004, does not explicitly distinguish ground motion selection for LTHA and NTHA. Actually, LTHA is not explicitly mentioned amongst the possible methods of seismic analysis; therefore, the few available indications provided about the ground motions selection are generally referred to NTHA.

Table 1. Comparison between EC8 and FEMA P-1050 provisions on input selection.

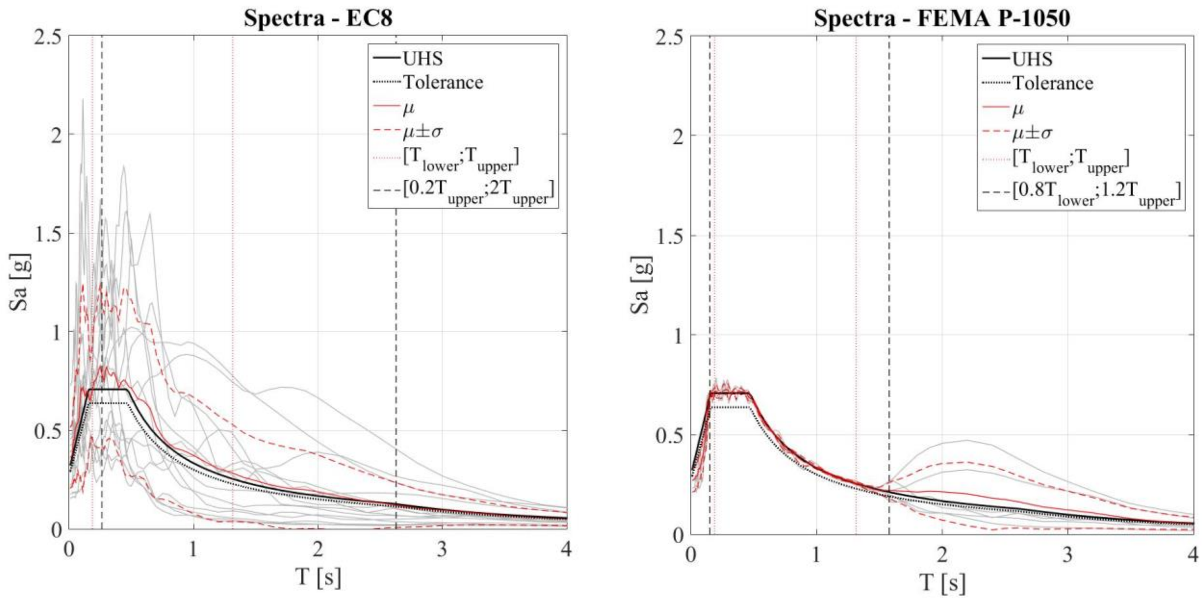
Requirement	EC8	FEMAP-1050/2015
Structural model	2D and/or 3D (swap of the two horizontal motion components is required).	3D (swap of the two horizontal motion components is required).
Number of ground motions	Minimum of 3 and – if < 7 the envelope of the responses should be used as design value; – if ≥ 7 the average of the responses should be used as design value.	– 3 only and the envelope of the responses should be used as design value, if spectral matching is utilized; – differently, not less than 11.
Matching tolerance	The average spectrum should not be less than 90% of the elastic UHS in the range of periods $[0.2T_1; 2T_1]$ and the mean of the spectral acceleration at T =0 should be ≥ $a_g S$ at the site.	The average spectrum of the matched motions should not be less than 90% of the elastic spectrum in the range of periods $\left[0.8T_{lower}; 1.2T_{upper}\right]$* and *response spectrum matching* is required.

* T_{upper} and T_{lower} are the first-mode period of vibration (T_1) and the one required for the structural model to reach the 90% of modal mass participation in each orthogonal direction.

Ground motion suites for the case study

Real ground motions are easily available from most common ground motion databases such as the European Strong Motion Database (ESMD), the NGA West2, etc. They are also preferable compared to artificial and simulated ones, thanks to the real frequency content, the correct time correlation between the components and the realistic energetic content referred to seismological parameters. A suite of seven unscaled real motions is selected from the ESMD and to be spectrum-compatible with the elastic UHS for LS-LS, through REXEL [17]. Such selection is the same considered in [1] and it matches the UHS between 10% lower and 30% upper tolerances over the period range of $[0; 4]$ s. The details on the record selected are reported in [1].

From this set, a sub-suite made by three pairs of ground motions is selected to be compatible with the FEMA P-1050/2015 provisions (in particular waveform IDs 196, 291 and 535 in ESMD). *Response spectrum matching* is performed through the Spectrum Matching Toolkit [18]. A range of periods equal to $[0.8T_{10}; 1.2T_1]$ is considered adequate for the benchmark building to cover the higher-modes effects. Figure 1 shows the spectrum matching according to the EC8 and FEMA P-1050. The RMSE for each pair of motions is calculated over the range of periods which effectively contribute in the analyses (see Table 2). These values are then compared with the results obtained from the LTHA analysis performed for the two suites of motions.



(a) (b)

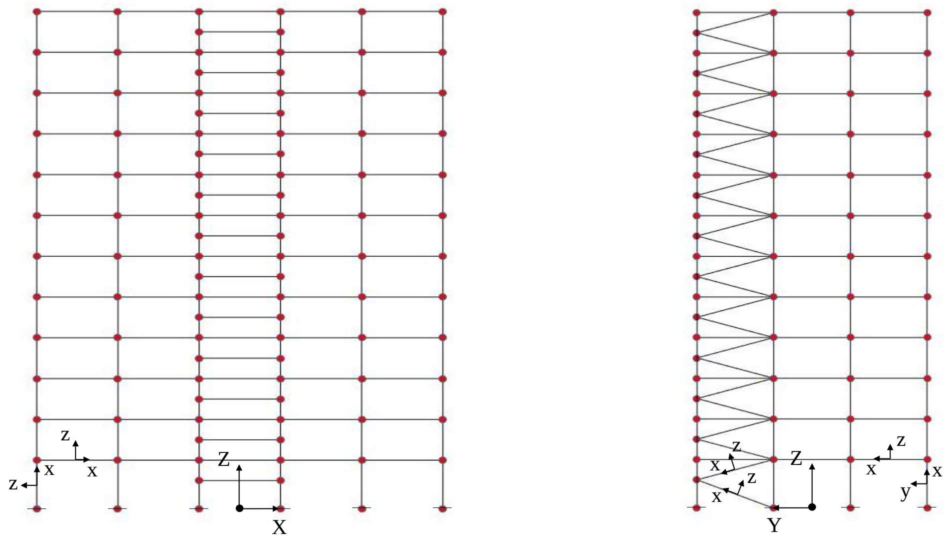
Figure 1. Matching of the response spectra for the considered ground motions suites: a) 7 pairs according to EC8 and b) 3 pairs according to FEMA P-1050/2015.

Table 2. Values of the Root-Mean-Square-Error (RMSE) for each component of the seven (EC8) and three (FEMA P-1050) pairs of the selected suites.

RMSE – EC8							
Matching range	**196**	**239**	**291**	**535**	**4673**	**6328**	**6334**
$[T_{10};T_1]$	0.6055 0.2438	0.3191 0.3029	0.2834 0.2110	0.2752 0.4421	0.2867 0.4228	0.4141 0.3133	0.1332 0.3706
RMSE – FEMA P-1050/2015							
Matching range	**196***	**-**	**291***	**535***	**-**	**-**	**-**
$[T_{10};T_1]$	0.0171 0.0146	- -	0.0241 0.0123	0.0229 0.0111	- -	- -	- -

Modelling description and proposed EC8 procedure for LTHA design

In order to run an elevated number of THA and to perform an automatic post-process check of the analyses results, an in-house Matlab-OpenSees code is developed. The elastic model of the benchmark structure is built up using *elasticBeamColumn* elements which account of flexural stiffness reduction due to cracks (50% of flexural moment of inertia) and torsional constant value typical of rectangular cross section [19]. No shear deformability is accounted herein, contrarily to the model considered in [1]. Also, for the sake of modelling simplicity, the staircase beams are modelled through only diagonal beams in lieu of “classic” knee beams (i.e., multi-linear beams with two staggered horizontal ends connected by an inclined beam) [20], see Figure 2. Storey diaphragms are assigned to each floor.



(a) (b)

Figure 2. Structural model of the benchmark building (a) XZ plane view (b) YZ plane view.

The first ten modes are considered for the benchmark building to capture the higher-modes contributions on the total response. Table 3 reports the values of the first ten periods of vibration of the structure.

Table 3. Periods of vibration of the benchmark building.

Mode	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
T [s]	1.31	1.19	1.13	0.47	0.40	0.40	0.27	0.23	0.22	0.19

Damping model

The viscous damping matrix is built up in compliance with FEMA P-1050/2015 provisions employing superposition of modal damping matrices which completely eliminates the “spurious” damping forces developed by Rayleigh damping model in NTHA as shown in [21]. In the case of LTHA the difference between the two methods is attributable to the assumption in Rayleigh model of user-defined damping ratios for only two periods while the damping ratios at other periods depend on the mass and stiffness proportional constants. This may lead to a different response when higher modes are significant. A viscous damping ratio equal to 5% is adopted for each mode because it is equal to the damping used in the development of the RSA.

Ground motions application and behavior factor

Thanks to the linearity of the problem, each ground motion component can be applied independently in each horizontal direction X and Y. At this stage, the behavior factor q can be applied by dividing the ground motion by q as per Eq. (2), where $S_{ae}(T_i)$ and $S_{ad}(T_i)$ are the values of the spectral acceleration corresponding to T_i evaluated on the elastic and design UHS, respectively [1].

$$q_{LTHA} = \min\left[\frac{S_{ae}(T_i)}{S_{ad}(T_i)}\right]_i \qquad \forall\, T_B \le T_i \le T_1 \tag{2}$$

The scaling factor is estimated to be equal to 1/4.86 = 0.206. For the selected EC8-compliant suite of motions, the number of linear transient analyses to perform is equal to 2x2x7=28 (2 horizontal components for each earthquake, 2 possible swaps along the horizontal directions X and Y, 7 pairs of ground motions). For the selected FEMA P-1050/2015-compliant suite of motions it results in 2x2x3=12 linear transient analyses to be performed (note that accidental eccentricity was not considered at this stage for the comparison).

P-Delta effects

P-Delta effects at LS-LS can be quantified from the unidirectional response of the structure through the evaluation of the interstorey drift sensitivity coefficient ϑ. Contrarily to FEMA P-1050/2015 for which this coefficient is evaluated through an additional static analysis, in our proposed procedure such coefficient can be evaluated in correspondence of the maximum displacement achieved over the time (amplified by *q* for the equal displacement rule when $T_1 \ge T_c$) for each unidirectional response belonging to the same earthquake. One value equal to the maximum value between direction X and Y among the storeys is considered for each earthquake. Then the amplification factor can be applied *a posteriori* when performing the combinations of the unidirectional responses for each earthquake. The amplification factors evaluated for the two input selections are reported in Table 4. For RSA the corresponding value is equal to 1.14.

Table 4. Values of the amplification factor accounting for P-Delta effects.

Suite of motions	**196**	**239**	**291**	**535**	**4673**	**6328**	**6334**
EC8	1.12	1.00	1.12	1.12	1.11	1.11	1.12
	196*	**-**	**291***	**535***	-	-	-
FEMA P-1050	1.11	-	1.12	1.12	-	-	-

Combination of the unidirectional time-history responses

Once the unidirectional responses are obtained and modified to account of the P-Delta effects, they can be combined with the gravity loads in order to evaluate the most unfavorable effects (local forces, displacements, reactions, etc). For each earthquake (i.e., pair of records), this procedure results in eight possible combinations (±X±Y) of the horizontal motion components if accidental eccentricity is neglected. Accidental eccentricity accounting for multiple sources of torsion (unconsidered mass distributions, uncertainties related to actual strength and stiffness, spatial variations in the ground excitation, etc.) is not considered at this stage. If accidental eccentricity is accounted through the shift of the center of mass, according to FEMA P-1050/2015, it results in 8x5 = 40 (4 shifted positions of the center of mass and one without shift) possible combinations. For each earthquake, the envelope of the effects from the combinations above mentioned, both in terms of maximum and minimum values, has to be evaluated. Once the envelope is evaluated for each earthquake, the procedure is different between FEMA P-1050/2015 and EC8: for the first these values are the design ones that need to be considered, while for the second these values need to be averaged in order to get the design ones.

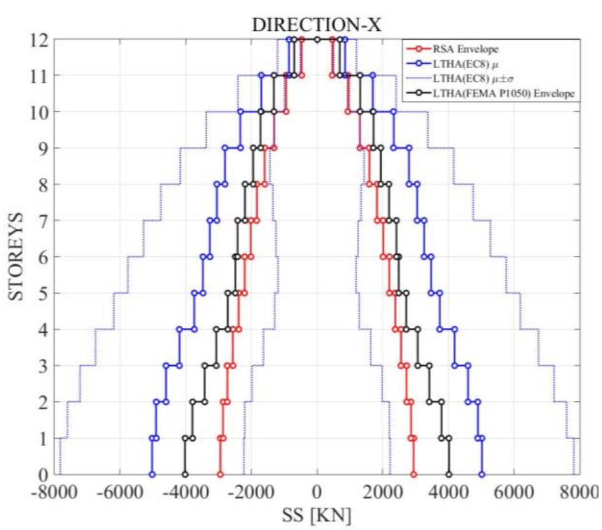
Acceptance criteria

One of the benefits of performing LTHA is the possibility to calculate Demand/Capacity (D/C) ratios of effects (bending moments, shears, interstorey drifts ratios) step by step for each combination by accounting of the real interaction between bending moments and axial force. According to the EC8 procedure proposed herein, the maximum value of D/C over the time is evaluated and enveloped among the earthquakes belonging to the same suite of ground motions. Subsequently, the average of the maximum values of D/C is evaluated and employed to double check the preliminary dimensioning.

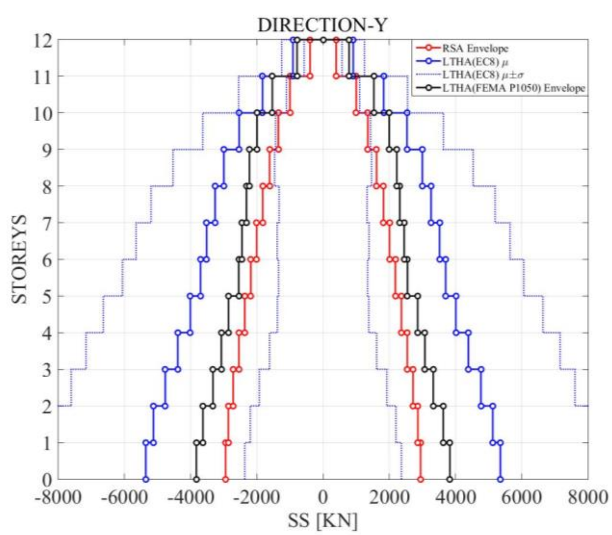
Performance Comparisons: RSA vs LTHA

In this section a comparison between the design results for RSA and LTHA is shown. LTHA results are shown for the case of input selection carried out according to EC8 and FEMA P-1050/2015. At this stage of the study, comparisons are shown for global response parameters: in terms of Storey Shears (SS) in Figure 4a and 3b and maximum Interstorey Drift Ratios (mIDR) in Figure 4a and 4b.

For the EC8-compliant LTHA design, results are shown in terms of average (μ) and standard deviation (σ). The results in terms of SS do not provide any unexpected results being in analogy with the trend predictable on the basis of the RMSE. The mIDR results in X direction provide an example underestimation in the drift at mid-storeys when the FEMA P-1050/2015 spectral matching procedure.



(a)



(b)

Figure 3. Comparison of SS in (a) X and (b) Y direction for the considered 12-storey RC MRF archetype building.

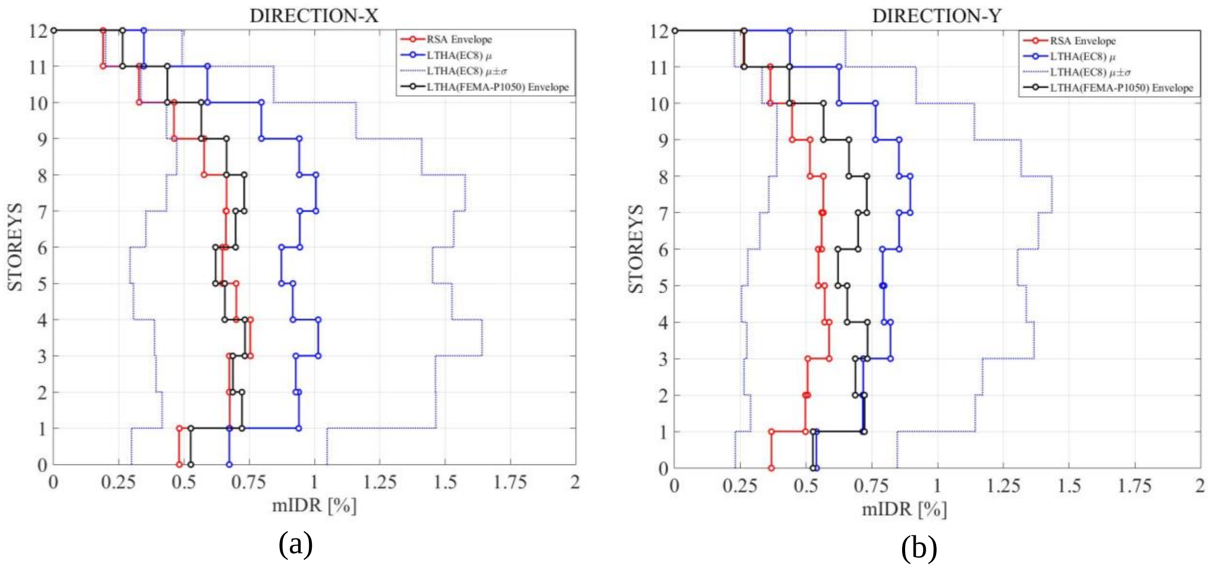


Figure 4. Comparison of mIDR in (a) X and (b) Y direction for the considered 12-storey RC MRF archetype building.

Conclusions

A new procedure for LTHA design according to EC8 is discussed. The current version of the EC8 was issued in 2004 and does not explicitly include LTHA as an option for force-based linear analysis. Recently, a new procedure has been included in FEMA P-1050/2015 and ASCE/SEI 7-16 which differs from the proposed one in some respects; such as input selection, behavior factor (or strength reduction factor) and P-Delta estimation. In this preliminary application, the conventional EC8 linear spectral scaling of seven pair of records is compared with spectral matching according to FEMA P-1050/2015.

The results emphasize some of the cons of using a very close matching when displacement-related quantities are of concern (e.g., mIDR); such us a lower response with respect to Response Spectrum Analysis results. On the other hand, the highly dispersed results of the EC8 approach need to be controlled with an *ad hoc* record-selection for Linear Time-History Analysis offering a reasonable compromise between accuracy for a preliminary estimation of record-to-record variability and effectiveness of design.

References

1. De Luca F, Lombardi L. EC8 design through Linear Time-History Analysis versus Response Spectrum Analysis – Is it an enhancement for PBEE? *16th World Conference on Earthquake (16WCEE 2017)*. Santiago, Chile, 2017.
2. Chopra AK. *Dynamics of the Structures – Theory and Applications to Earthquake Engineering*. Prentice Hall: 2012.
3. Haselton CB, Fry A, Hamburger RO, Baker JW, Zimmerman RB, Luco N, Elwood KJ, Hooper JD, Charney FA, Pekelnicky RG, Whittaker A. Response History Analysis for the Design of New Buildings in the NEHRP Provisions and ASCE/SEI 7 Standard: Part I – Overview and Specification of Ground Motions. *Earthquake*

Spectra 2017; **33** (2): 373-395.

4. Haselton CB, Fry A, Hamburger RO, Baker JW, Zimmerman RB, Luco N, Elwood KJ, Hooper JD, Charney FA, Pekelnicky RG, Whittaker A. Response History Analysis for the Design of New Buildings in the NEHRP Provisions and ASCE/SEI 7 Standard: Part II – Structural Analysis Procedures and Acceptance Criteria. *Earthquake Spectra* 2017; **33** (2): 397-417.
5. Fragiadakis M, Vamvatsikos D, Aschheim M. Application of Nonlinear Static Procedures for seismic assessment of regular RC Moment Frame Buildings. *Earthquake Spectra* 2014; **30** (2): 767-794.
6. Charney F. A new Linear Response History Analysis Procedure for the 2015 NEHRP Recommended Provisions and for ASCE 7-16. *Structures Congress 2015*. Portland, Oregon, 2015.
7. Federal Emergency Management Agency (FEMA). *FEMA P-1-5-1 (2015): NEHRP Recommended Seismic Provisions for New Buildings and Other Structures – Volume I*. Washington, D.C.
8. American Society of Civil Engineers (ASCE). *ASCE/SEI 7-16 (2017): Minimum Design Loads and Associated Criteria for Buildings and Other Structures*. Reston, VA.
9. Open System for Earthquake Engineering Simulation, OpenSees (2006), v2.5.0 Rev 6248 64-Bit, Pacific Earthquake Engineering Research Center (PEER), University of California, Berkeley, CA. Available at http://opensees.berkeley.edu/
10. De Luca F, Verderame GM. The accuracy of CQC and response spectrum analysis in the case of impulsive earthquakes. *Proceeding of the 11th International Conference on Structural Safety and Reliability (ICOSSAR 2013)*. New York, 2013.
11. European Committee for Standardization (CEN). *EN 1998-1 Eurocode 8 (2004): Design of structures for earthquake resistance – Part 1: General rules, seismic actions and rules for buildings*. Brussels, BE.
12. European Committee for Standardization (CEN). *EN 1992-1-1 Eurocode 2 (2004): Design of concrete structures – Part 1-1: General rules and rules for buildings*. Brussels, BE.
13. National Institute of Standards and Technology (NIST). NIST GCR 11-917-15: Selecting and Scaling Earthquake Ground Motions for Performing Response History Analyses. Prepared by the NEHRP Consultants Joint Venture for the NIST, Gaithersburg, MD.
14. Hancock J, Bommer JJ, Stafford PJ. Numbers of scaled and matched accelerograms required for inelastic dynamic analyses. *Earthquake Engineering & Structural Dynamics* 2008; **37** (14): 1585-1607.
15. Bazzurro P, Luco N. Do scaled and spectrum-matched near-source records produce biased nonlinear structural responses? *Proceedings of the 8th National Conference on Earthquake Engineering (8NCEE 2006).* San Francisco, California, 2006.
16. Shahi SK. *A probabilistic framework to include the effect of near-fault directivity in seismic hazard assessment*. Ph.D. Thesis, Stanford University, Stanford, CA, 2013. Pulse classification available at https://web.stanford.edu/~bakerjw/pulse_classification_v2/Pulse-like-records.html
17. Iervolino I, Galasso C, Cosenza E. REXEL: computer aided record selection for code-based seismic structural analysis. *Bulletin of Earthquake Engineering* 2010; **8**: 339-362. REXEL is available at http://www.reluis.it/index.php?option=com_content&view=article&id=118&Itemid=&lang=en
18. Jayamon JR, Charney FA. Multiple Ground Motion Spectrum Match Tool for Use in Response History Analysis. *Structures Congress 2015*. Portland, Oregon, 2015.
19. Young WC, Budynas RG. *Roark's Formulas for Stress and Strain.* McGraw-Hill: 2002.
20. Fardis MN. *Seismic Design, Assessment and Retrofitting of Concrete Buildings – Based on EN-Eurocode 8*. Springer: 2009.
21. Chopra AK, McKenna F. Modeling Viscous Damping in Nonlinear Response History Analysis of Buildings for Earthquake Excitation. *Earthquake Engineering and Structural Dynamics* 2016; **45**: 193-211.